Central University of Punjab

Course Scheme & Syllabus
for
Ph. D. Course Work
in
Mathematics

Department of Mathematics and Statistics
School of Basic Sciences

(Batch: 2026-27)

Structure of Course work for PhD in Mathematics

Students can move into the Ph.D. programme after successful completion of Ph. D. Course work during the first two semesters, provided they meet the requirements specified by the university.

Graduate Attributes:

Learners will be able to develop a broad understanding of recent theories, tools and techniques in research areas of Mathematics. They will be able to be competent to apply various Mathematical techniques in a variety of situations. They will be able to independently develop and plan research in various areas of mathematics and allied areas.

Semester I					
Subject Code	**Subject Name**	**Credit Hours**			**Course Credits**
		L	**T**	**P**	
MAT.701	Research Methodology	2	0	0	2
MAT.702	Computer Applications	2	0	0	2
MAT.751	Research and Publication Ethics	2	0	0	2
MAT.752	Teaching Assistantship	0	0	2	1
UNI.753	Curriculum, Pedagogy and Evaluation	1	0	0	1
Opt any two out of the following					
MAT.704	Symmetries and Differential Equations	4	0	0	4
MAT.705	Fractional Calculus	4	0	0	4
MAT.706	Advanced Partial Differential Equations	4	0	0	4
MAT.707	Differential Equations and Boundary-Value Problems	4	0	0	4
MAT.708	Differentiable Manifolds	4	0	0	4
MAT.709	Algebraic Topology	4	0	0	4
MAT.710	Riemannian Geometry	4	0	0	4
MAT.711	Riemann - Finsler Geometry	4	0	0	4
MAT.712	General Relativity	4	0	0	4
MAT.713	Mathematical Modelling in Biological Systems	4	0	0	4
MAT.714	Qualitative Theory of Dynamical Systems	4	0	0	4

MAT.715	Advance Linear Algebra	4	0	0	4
MAT.718	Stochastic Differential Equations	4	0	0	4
MAT.719	Fluid Dynamics	4	0	0	4
MAT.720	Advanced Topics in Topology and its Applications	4	0	0	4
MAT.721	Stochastic Processes & its Applications	4	0	0	4
MAT.722	Mathematical Control Theory	4	0	0	4
MAT.723	Lie Algebras	4	0	0	4
MAT.724	Game Theory & its Applications	4	0	0	4
MAT.725	Complex Manifolds	4	0	0	4
MAT.726	Contact Manifolds	4	0	0	4
				Total Credits	**16**

Note: The students will be suggested to read recently published research articles to update their knowledge in the area of their interest or suggested by the supervisor.

Course Title: Research Methodology
Course Code: STA.701
Total Hours: 30

L	T	P	Credits
2	0	0	2

Learning outcomes:

The student will be able to:

CLO1: Understand meaning, objectives, characteristics, significance, and types of research.

CLO2: Understand the different steps of formulation of research problems.

CLO3: Use latex to write different types of reports.

CLO4: Understand the basics of different bibliography/reference preparation styles.

UNIT/Hours	Content	Mapping with CLO
Unit-I /8 Hours	**Introduction:** Meaning, Objectives, Characteristics, Significance, and Types of Research. **Formulating Research Problem:** Understanding a Research Problem, Selecting the Research Problem, Steps in Formulation of a Research Problem, Formulation of Research Objectives, and Construction of Hypothesis.	**CLO1** **CLO2**
Unit-II/ 8 Hours	Installation of the software LaTeX, Understanding LaTeX compilation and LaTeX editors, Basic syntax, Writing mathematical equations, Matrices, Tables, Inclusion of graphics into LaTeX file. **Page configurations:** Title, Abstract, Keywords, Chapter, Sections and Subsections, References and their citations, Labeling of equations, Table of contents, List of figures, List of tables, Page numbering, Generating index.	**CLO3**
Unit-III/7 Hours	**Packages:** amsmath, amssymb, amsthm, amsfonts, hyperrefer, graphic, color, xypic, latexsym, natbib, setspace, multicol, subcaption, url, verbatim, tikz, and geometry. **Classes:** Article, Report, Bresearchook, Letter, Slides, Beamer.	**CLO3**

Unit-IV/7 Hours	**Report Writing:** Types of Reports – Technical and Popular Reports, Significance of Report Writing, Different Steps in Writing Report, Art of Writing Research Proposals, Research Papers, Project Reports, and Dissertations/Thesis; Basics of Citation and Bibliography/Reference Preparation Styles; Report Presentation: Oral and Poster Presentations of Research Reports. **TRANSACTION MODE**: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.	**CLO4**

Suggested Readings:

1. Kothari, C.R. and G. Garg (2014): *Research Methodology*: Methods and Techniques, 3rd ed., New Age International Pvt. Ltd. Publisher
2. Kumar, R. (2014): *Research Methodology - A Step-By-Step Guide for Beginners*, 4th ed., Sage Publications
3. J. Anderson, *Thesis and Assignment Writing*, 4th ed., Wiley, USA, 2001.
4. Catherine Dawson, *Practical Research Methods,* New Delhi, UBS Publishers' Distributors, 2014.
5. L. Lamport. LATEX: A Document Preparation System, User's Guide and Reference Manual. 2nd Edition, Addison Wesley, New York, 1994.
6. Copyright Protection in India [website: http:copyright.gov.in].
7. World Trade Organization [website: www.wto.org].

Course Title: Computer Applications

Course Code: STA.702

Total Hours: 30

L	T	P	Credits
2	0	0	2

Learning outcomes:

The student will be able to:

CLO1: Use different operating systems and their tools easily.

CLO2: Use word processing software, presentation software, spreadsheet software.

CLO3: Use computers in every field like teaching, industry and research.

UNIT/Hours	Content	Mapping with CLO
UNIT I/8 Hours	Computer Fundamentals: Introduction to Computer, Input devices, Output Devices, Memory (Primary and Secondary), Concept of Hardware and Software, C.P.U., System bus, Motherboard, Ports and Interfaces, Expansion Cards, Ribbon Cables, Memory Chips, Processors, Software: Types of Software, Operating System, User Interface of popular Operating System, Introduction to programming language, Types of Computer.	CLO1
UNIT Hours II/7	Computer Network: Introduction to Computer Network, Types of Network: LAN, WAN and MAN, Topologies of Network, Internet concept, WWW. Word Processing: Text creation and Manipulation; Table handling; Spell check, Hyper-linking, Creating Table of Contents and table of figures, Creating and tracking comments, language setting and thesaurus, Header and Footer, Mail Merge, Different views, Creating equations, Page setting, Printing, Shortcut keys.	CLO2
UNIT III/8 Hours	Presentation Tool: Creating Presentations, Presentation views, working on Slide Transition, Making Notes Pages and Handouts, Drawing and Working with Objects, Using Animations, Running and Controlling a Slide Show, Printing Presentations, and Shortcut keys. Spreadsheet: Entering and editing data in cell, Basic formulas and functions, deleting or inserting cells, deleting or inserting rows and columns, printing of SpreadSheet, Shortcut keys.	CLO3
UNIT Hours	Use of Computers in Education and Research: Data analysis tools, e-Library, Search engines	CLO4

IV/7	related to research, Research paper editing tools like Latex. Transactional Modes: PPT Video e-content google drive	

Suggested Readings:

1. Sinha, P.K. Computer Fundamentals. BPB Publications.
2. Goel, A., Ray, S. K. 2012. Computers: Basics and Applications. Pearson Education India.
3. Microsoft Office Professional 2013 Step by Step https://ptgmedia.pearsoncmg.com/images/9780735669413/samplepages/97807356 694 13.pdf

Course Title: Research and Publication Ethics Course
Code: MAT.751
Total Hours: 30

Learning outcomes:
The student will be able to:
CLO1: Understand the philosophy and value of publication ethics.
CLO2: Understand ethics with respect to science and research.
CLO3: Identify the predatory publications and open access publications.
CLO4: Use different software and their tools to check plagiarism.

UNIT/Hours	Content	Mapping with CLO
I **8 Hours**	PHILOSOPHY AND ETHICS: Introduction to philosophy definition, nature and scope, concept, branches; Ethics definition, moral of moral judgements and reactions. SCIENTIFIC CONDUCT: Ethics with respect to science and research; Intellectual honesty and research integrity; Scientific misconducts Falsification, Fabrication, and plagiarism (FFP); Redundant publication duplicate and overlapping publication, salami slicing; Selective reporting and misrepresentation of data	**CLO1**
II **7 Hours**	PUBLICATION ETHICS: Publication ethics definition introduction and importance; Best practices / standards setting initiatives and guidelines: COPE, WAME, etc.; Conflicts of interest; Publication misconduct: definition, concept, problems that lead to unethical behaviour and vice versa, types; violation of publication ethics, authorship and contributorship; identification of publication misconduct, complaints and appeals; predatory publishers and journals	**CLO2**
III **8 Hours**	OPEN ACCESS PUBLISHING: Open access publication and initiatives; SHERPA/RoMEO online resource to check publisher copyright & self-archiving policies; software tool to identify predatory publications developed by SPPU; journal finder/ journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc. PUBLICATION MISCONDUCT:	**CLO3**

	A. Group Discussions: a) Subject specific ethical issues, FFP, authorship; b) Conflicts of interest; c) Complaints and appeals: examples and fraud from India and abroad B. Software tools: Use of plagiarism software like Turnitin, Urkund and other open source software tools.	
IV **7 Hours**	DATABASES AND RESEARCH METRICS: A. Databases: a) Indexing databases; b) Citation databases: web of Science, Scopus, etc. B. Research Metrics: a) Impact Factor of journal as per journal Citation Report, SNIP, SJR, IPP, Cite Score b) Metrics, h-index, g-index, i10 index, altmetrics	**CLO4**

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E- team teaching/Self-learning/Practical.

Suggested Readings:

1. Melville, S., and Goddad, W. (1996). Research Methodology: An Introduction to Science and Engineering students. South Africa: Juta Academic.
2. Kothari, C.R. and G. Garg (2014): *Research Methodology*: Methods and Techniques, 3rd ed., New Age International Pvt. Ltd. Publisher

Course Title: Research and Publication Ethics Course
Code: MAT.751
Total Hours: 30

Learning outcomes:
The student will be able to:
CLO1: Understand the philosophy and value of publication ethics.
CLO2: Understand ethics with respect to science and research.
CLO3: Identify the predatory publications and open access publications.
CLO4: Use different software and their tools to check plagiarism.

UNIT/Hours	Content	Mapping with CLO
I **8 Hours**	PHILOSOPHY AND ETHICS: Introduction to philosophy definition, nature and scope, concept, branches; Ethics definition, moral of moral judgements and reactions. SCIENTIFIC CONDUCT: Ethics with respect to science and research; Intellectual honesty and research integrity; Scientific misconducts Falsification, Fabrication, and plagiarism (FFP); Redundant publication duplicate and overlapping publication, salami slicing; Selective reporting and misrepresentation of data	**CLO1**
II **7 Hours**	PUBLICATION ETHICS: Publication ethics definition introduction and importance; Best practices / standards setting initiatives and guidelines: COPE, WAME, etc.; Conflicts of interest; Publication misconduct: definition, concept, problems that lead to unethical behaviour and vice versa, types; violation of publication ethics, authorship and contributorship; identification of publication misconduct, complaints and appeals; predatory publishers and journals	**CLO2**
III **8 Hours**	OPEN ACCESS PUBLISHING: Open access publication and initiatives; SHERPA/RoMEO online resource to check publisher copyright & self-archiving policies; software tool to identify predatory publications developed by SPPU; journal finder/	**CLO3**

	journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc. PUBLICATION MISCONDUCT: A. Group Discussions: a) Subject specific ethical issues, FFP, authorship; b) Conflicts of interest; c) Complaints and appeals: examples and fraud from India and abroad B. Software tools: Use of plagiarism software like Turnitin, Urkund and other open source software tools.	
IV **7 Hours**	DATABASES AND RESEARCH METRICS: A. Databases: a) Indexing databases; b) Citation databases: web of Science, Scopus, etc. B. Research Metrics: a) Impact Factor of journal as per journal Citation Report, SNIP, SJR, IPP, Cite Score b) Metrics, h-index, g-index, i10 index, altmetrics	**CLO4**

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E- team teaching/Self-learning/Practical.

Suggested Readings:

1. Melville, S., and Goddad, W. (1996). Research Methodology: An Introduction to Science and Engineering students. South Africa: Juta Academic.
2. Kothari, C.R. and G. Garg (2014): *Research Methodology*: Methods and Techniques, 3rd ed., New Age International Pvt. Ltd. Publisher

Course Name

Course Code: STA.752
Total Hours: 30

L	T	P	Credits
0	0	2	1

Learning Outcome:

At the end of this skill development course, the scholars shall be able to

CLO 1: familiarize themselves with the pedagogical practices of effective classroom delivery and knowledge evaluation system

CLO 2: manage large and small classes using appropriate pedagogical techniques for different types of content

Activities and Evaluation:

- The scholars shall attend Master degree classes of his/her supervisor to observe the various transaction modes that the supervisor follows in the classroom delivery or transaction process one period per week.
- The scholars shall be assigned one period per week under the direct supervision of his/her supervisor to teach the Master degree students adopting appropriate teaching strategy(s).
- The scholars shall be involved in the examination and evaluation system of the Master degree students such as preparation of questions, conduct of examination and preparation of results under the direction of the supervisor.
- At the end of the semester, the supervisor shall conduct an examination of teaching skills learned by the scholar as per the following **evaluation criteria**:
    - The scholars shall be given a topic relevant to the Master degree course of the current semester as his/her specialization to prepare lessons and deliver in the classroom before the master degree students for one hour (45 minutes teaching + 15 minutes interaction).
    - The scholars shall be evaluated for a total of 50 marks comprising *content knowledge* (10 marks), *explanation and demonstration skills* (10 marks), *communication skills* (10 marks), *teaching techniques employed* (10 marks), and classroom interactions (10).

Course Title: CURRICULUM, PEDAGOGY AND EVALUATION

Course Code: UNIV.753

Total Hours: 15

L	T	P	Credits
1	0	0	1

Learning outcomes:

After completion of the course, scholars shall be able to

CLO1: learn different types of Curriculum Designs.

CLO2: learn the process of Curriculum Development.

CLO3: develop the skills of adopting innovative pedagogies and conducting students' assessment.

CLO4: develop curriculum of a specific course/programme.

UNIT/Hours	Content	Mapping with CLO
I **4 Hours**	**Bases and Principles of Curriculum** 1. Curriculum: Concept and Principles of curriculum development, Foundations of Curriculum Development. Types of Curriculum Designs- Subject centered, learner centered, experience centered and core curriculum. Designing local, national, regional and global specific curriculum. Choice Based Credit System and its implementation.	**CLO1**
II **4 Hours**	**Curriculum Development** 3. Process of Curriculum Development: Formulation of graduate attributes, course/learning outcomes, content selection, organization of content and learning experiences, transaction process. Comparison among Interdisciplinary, multidisciplinary and trans-disciplinary approaches to curriculum.	**CLO2**
III **3 Hours**	**Curriculum and Pedagogy** 5. Conceptual understanding of Pedagogy. 6. Pedagogies: Peeragogy, Cybergogy and Heutagogy with special emphasis on Blended learning, Flipped learning, Dialogue, cooperative and collaborative learning Three e- techniques: Moodle, Edmodo, Google classroom	**CLO3**
IV **4 Hours**	**Learners' Assessment**	**CLO4**

	8. Assessment Preparation: Concept, purpose, and principles of preparing objective and subjective questions. 9. Conducting Assessment: Modes of conducting assessment – offline and online; use of ICT in conducting assessments. Evaluation: Formative and Summative assessments, Outcome based assessment, and scoring criteria.	

Transaction Mode

Lecture, dialogue, peer group discussion, workshop

Evaluation criteria

There shall be an end term evaluation of the course for 50 marks for a duration of 2 hours. The course coordinator shall conduct the evaluation.

- Allyn, B., Beane, J. A., Conrad, E. P., & Samuel J. A., (1986). *Curriculum Planning and Development*. Boston: Allyn & Bacon.
- Brady, L. (1995). *Curriculum Development*. Prentice Hall: Delhi. National Council of Educational Research and Training.
- Deng, Z. (2007). Knowing the subject matter of science curriculum, *Journal of Curriculum Studies*, 39(5), 503-535. https://doi.org/10.1080/00220270701305362
- Gronlund, N. E. & Linn, R. L. (2003). *Measurement and Assessment in teaching*. Singapore: Pearson Education
- McNeil, J. D. (1990). *Curriculum: A Comprehensive Introduction*, London: Scott, Foreman/Little
- Nehru, R. S. S. (2015). *Principles of Curriculum*. New Delhi: APH Publishing Corporation.
- Oliva, P. F. (2001). *Developing the curriculum* (Fifth Ed.). New York, NY: Longman
- Stein, J. and Graham, C. (2014). *Essentials for Blended Learning: A Standards-Based Guide*. New York, NY: Routledge.

Web Resources

- https://www.westernsydney.edu.au/ data/assets/pdf_file/0004/467095/Fundamentals_of_Blended_Learning.pdf
- https://www.uhd.edu/academics/university-college/centers-offices/teaching-learning-excellence/Pages/Principles-of-a-Flipped-Classroom.aspx
- http://leerwegdialoog.nl/wp-content/uploads/2018/06/180621-Article-The-Basic-Principles-of-Dialogue-by-Renate-van-der-Veen-and-Olga-Plokhooij.pdf

Course Title: Symmetries and Differential Equations

Course Code: MAT.704

Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:.

The student will be able to:

CLO1. know the basics of dimensional analysis and Lie group of transformations.

CLO2. Learn the basic applications of the Lie group of transformation for solving ordinary and partial differential equations.

CLO3. Learn the concept of extended infinitesimal transformations.

CLO4. Understand the basic concept of multi-parameter Lie groups of transformations.

UNIT/Hours	Content	Mapping with CLO
I **15 Hours**	Dimensional Analysis: Buckingham Pi-theorem, Assumptions behind dimensional analysis, Conclusions from dimensional analysis, Proof of the Buckingham Pi-theorem and examples, Application of dimensional analysis to partial differential equations, Generalization of dimensional analysis, Invariance of partial differential equations under scaling of variables	**CLO1**
II **15 Hours**	Lie Group of Transformations: Groups, Examples of groups, Groups of transformations, One-parameter Lie group of transformations, Examples of one-parameter Lie groups of transformations, Infinitesimal transformations: First fundamental theorem of Lie, Infinitesimal generators, Invariant functions.	**CLO2**
III **15 Hours**	Canonical coordinates, Invariant surfaces, Invariant curves, Invariant points, Extended transformations: Extended group transformations-one dependent and one independent variable, Extended infinitesimal transformations-one dependent and one independent variable, Extended transformations-one dependent and n independent variables.	**CLO3**
IV **15 Hours**	Multi-parameter Lie groups of transformations; Lie algebras, r-parameter Lie groups of transformations, Lie algebras, Examples of Lie algebras, Solvable Lie algebras.	**CLO4**

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. G. W. Bluman and A. C. Anco, *Symmetry and Integration Methods for Differential Equations*, Appl. Math. Sci., 154, Springer, New York, 2002.
2. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, Appl. Math.Sci., Springer-Berlin, 1989.
3. P. J. Olver, *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York 1993.
4. L. V. Ovsiannikov, *Group Properties of Differential Equations*, Novosibirsk, Moscow, 1962.

Course Title: Fractional Calculus

Course Code: MAT.705

Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:.
The student will be able to:
CLO1. Learn the basics of fractional calculus.
CLO2. Learn the different types of fractional derivatives.
CLO3. Understand the concept of linear fractional differential equations.
CLO4. Apply different techniques for solving fractional differential equations.

UNIT/Hours	Content	Mapping with CLO
I **15 Hours**	**Special Functions of Fractional Calculus:** Gamma function, Some properties of Gamma function, Beta function, Contour integral representation. Fractional derivatives and integrals, GrunwaldLetnikov Fractional derivatives, Riemann-Liouville fractional derivatives, Caputo's fractional derivative, The Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.	**CLO1**
II **15 Hours**	Sequential fractional derivatives. Left and right fractional derivatives. Properties of fractional derivatives. Laplace transforms of fractional derivatives. Fourier transforms of fractional derivatives. Mellin transforms of fractional derivatives.	**CLO2**
III **15 Hours**	**Linear Fractional Differential Equations:** Fractional differential equation of a general form. Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. The Laplace transform method. Standard	**CLO3**

	fractional differential equations. Sequential fractional differential equations.	
IV **15 Hours**	**Fractional Differential Equations:** Introduction, Linearly independent solutions, Solutions of the homogeneous equations, Solution of the non-homogeneous fractional differential equations, Reduction of fractional differential equations to ordinary differential equations. Semi differential equations.	**CLO4**

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. K. B. Oldham & J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Dover Publications Inc, 2006.
2. K. S. Miller & B. Ross., *An Introduction to the Fractional Calculus and Fractional Differential Equations Hardcover*, Wiley Blackwell, 1993.
3. I. Podlubny, *Fractional Differential Equations*, Academic Press, 1998.

Course Title: Advanced Partial Differential Equations-II

L	T	P	Credits
4	0	0	4

Course Code: MAT.706

Total Hours: 60

The students will be able to

CLO1. use smooth functions for approximations.

CLO2. understand the basic concepts of Weak solutions of elliptic boundary value problems

CLO3. study scientific evolution equations.

CLO4. learn the conditions for existence of minimizers.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	**Distribution**: Test functions and distributions, examples, operations on distributions, supports and singular supports, convolution, fundamental solutions, fourier transform, Schwartz space, tempered distributions. **Sobolev Spaces**: Basic properties, approximation by smooth functions, extension theorems, compactness theorems, dual spaces, functional order spaces, trace spaces, trace theory, inclusion theorem. **Activity:** Students will use smooth functions for approximations.	CLO1
II 15 Hours	**Weak solutions of elliptic boundary value problems**: variational problems, weak formulation of elliptic PDE, regularity, Galerkin method, Maximum principles, eigenvalue problems, Introduction to finite element methods. **Activity:** Students will obtain weak solutions of some elliptic boundary value problems.	CLO2
III	**Evolution Equations**: unbounded linear operators, C_0 – semigroups, Hille-Yosida theorem, contraction semigroup on	CLO3

15 Hours	Hilbert spaces, heat equation, wave equation, Schrödinger equation, inhomogeneous equations. **Activity:** Students will study scientific evolution equations.	
IV 15 Hours	**Calculus of Variations**: Euler-Lagrange equation, second variation, existence of minimizers (coactivity, lower semi-continuity, convexity), regularity, constraints (nonlinear eigenvalue problems, variational inequalities, harmonic maps, incompressibility), critical points (mountain pass theorem and applications to elliptic PDE). **Activity:** Students will check the conditions for existence of minimizers. .	CLO4

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Books:

1. S. Kesavan, *Topics in Functional Analysis and Application*, Wiley-Eastern, New International, New Delhi, 1999.
2. L. C. Evans, *Partial Differential Equations. Graduate Studies in Mathematics*, American Mathematical Society, 2nd Edition, Indian Reprint, 2014.
3. Rao, K. S., *Introduction to Partial Differential Equation*, 2nd Edition, PHI Learning Pvt. Ltd. 2010.
4. Amarnath, T., *An Elementary Course in Partial Differential Equations*, 2nd Edition, Narosa Publishing House 2012.
5. Sneddon, I. N., *Elements of Partial Differential Equations*, McGraw-Hill Book Company, New York 1988.

Course Title: Differential Equations and Boundary-Value Problems

L	T	P	Credits
4	0	0	4

Course Code: MAT.707

Total Hours: 60

Learning outcomes:.
The students will be able to
CLO1: Review the basic concepts of Existence and uniqueness of solutions of ODEs.
CLO2: Explain the stability and Liapunov functions for Non-linear system of ODE.
CLO3: Classify the First-order PDEs, into cauchy problems and second order PDEs into Parabolic, Hyperbolic and Elliptic.
CLO4: Review the basic concepts of Sobolev Spaces and the system of conservation laws.

Learning Outcomes: The objective of this course is to ensure that a student learns basic differential equations and boundary value problems.

Units/ Hours	**Contents**	**Mapping with CLO**
I 15 Hours	Existence and uniqueness of solutions of ODEs, power series solution, singular points, some special functions. Non-linear system of ODE: preliminary concepts and definitions, the fundamental existence-uniqueness results, dependence on initial conditions and parameters, the maximum interval of existence. **Activity:** Students will do some exercise on Existence and uniqueness of solutions of ODEs.	CLO1
II 15 Hours	Linearlization, stability and Liapunov functions, saddle, nodes, foci and centers, normal form theory and Hamiltonian systems. Boundary value problems: Green's function method, Sturm-Liouville problem. **Activity:** Students will explore the stability and Liapunov functions for Non-linear system of ODE.	CLO2
III 15 Hours	First-order PDEs, cauchy problem, method of characteristics, second-order PDEs, classification, characteristics and canonical forms. Elliptic	CLO3

	boundary value problems: maximum principle, Green's function. **Acitivity:** students will apply the First-order PDEs, into cauchy problems and second order PDEs into Parabolic, Hyperbolic and Elliptic.	
IV 15 Hours	Sobolev spaces, variational formulations, weak solutions, Lax-Milgram theorem, trace theorem, Poincarénequality, energy estimates, Fredholm alternative, regularity estimates, system of conservation laws, entropy criteria. **Acitivity:** Students will study the Sobolev Spaces and the system of conservation laws.	CLO4

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. L. Perko, *Differential Equations and Dynamical Systems*, 3rd Edition Springer-Verlag New York, 2001.
2. J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, 1st Edition, Springer-Verlag, New York, 1983.
3. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, 2nd Edition, Springer-Verlag, New York, 1990.
4. L. C. Evans, *Partial Differential Equations, Graduate Studies in Mathematics*, Vol. 19, American Mathematical Society, Providence, 1998.
5. R. C. McOwen, *Partial Differential Equations-Methods and Applications*, 2nd Edition Pearson Education Inc., Indian Reprint 2002.
6. S. J. Farlow, *Partial Differential Equations for Scientists and Engineers*, Dover Publications, New York, 1982.

Course Title: Differentiable Manifolds

L	T	P	Credits
4	0	0	4

Course Code: MAT.708

Total Hours: 60

Learning Outcomes:

The students will be able to

CLO1: learn the basics of Differentiable Manifolds.

CLO2: appreciate the topics covered in allied courses like Algebraic Topology, Riemannian geometry and Riemann-Finsler geometry.

CLO3: adequately prepared for pursuing research in above mentioned topics

CLO4: Understand the fundamentals of Tensor algebra.

UNIT/Hours	Content	Mapping with CLO
I 14 Hours	Topological and Smooth manifolds, Manifolds with boundary, Smooth functions on manifolds, Smooth maps between manifolds, Diffeomorphisms, Partition of Unity and Bump functions. Tangent vectors and tangent space, Differential of a smooth map.	CLO1 CLO2 CLO3
II 16 Hours	Curves on a manifold, Velocity vector of curves. Submersions, Immersions and Embeddings. Critical and regular points, Smooth covering maps. Submanifolds: Embedded submanifolds, Immersed submanifolds, Restricting maps to Submanifolds. Tangent space to submanifolds. Sard's theorem, The Whitney embedding theorem(statement only).	CLO1 CLO2 CLO3
III 16 Hours	Vector fields, Lie brackets, Integral curves, Orientation of manifolds. Topological groups, Lie groups, Lie group homomorphisms, Lie subgroups, Lie transformation groups, Left invariant vector fields of a Lie group, Lie Algebra of a Lie group. Group actions and Equivariant maps.	CLO1 CLO2 CLO3
IV 14 Hours	Differential forms on R^n and on a general manifold, Exterior algebra, Exterior product, Exterior derivative, Pull back of a map, Interior product and Lie derivative, Commutative vector fields and Frobenius theorem.	CLO1 CLO2 CLO3 CLO4

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, Academic Press, New York, 2007.
2. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pvt. Ltd., 2022.
3. L. Conlon, *Differentiable Manifolds*, 2nd edition, Birkhauser Boston, Cambridge, MA, 2011.
4. N. J. Hicks, *Notes of Differential Geometry*, D. Van Nostrand Reinhold Company, New York, 1965.
5. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics),* Hindustan Book Agency, 2002.
6. J. M. Lee, *Introduction to Smooth Manifolds*, GTM, Vol. 218, Springer, New York, 2021.
7. S. Morita, Geometry of Differential forms, Translations of Mathematical Monographs, Vol. 201, American Mathematical Society, 2001.
8. M. Spivak, A Comprehensive Introduction to Modern Differential Geometry Vol.1 Third Edition, Publish or Perish, INC. Houston, Texas 1999.
9. L. W. Tu, *An Introduction to Manifolds*, Second edition, Springer, 2011.

Course Title: Algebraic Topology
Course Code: MAT.709
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:
The students will be able to
CLO1: Understand the fundamental concept of Homotopy theory to pursue research.
CLO2: Develop the knowledge of fundamental groups and covering spaces with related properties.
CLO3: Construct homotopies and prove homotopy equivalence.
CLO4: Understand free products and free groups and their applications.
CLO5: Understand the classification of covering spaces.
CLO6: apply his or her knowledge of algebraic topology to formulate and solve problems of a geometrical and topological nature in mathematics.

UNIT/Hours	Content	Mapping with CLO
I **15 Hours**	The Fundamental group: Homotopy of paths, Homotopy classes, The Fundamental group, Change of base point, Topological invariance, Covering spaces, The Fundamental group of the circle.	**CLO1** **CLO2** **CLO3** **CLO6**
II **15 Hours**	Retractions and fixed points, No Retraction Theorem, The Fundamental theorem of Algebra, The Borsuk-Ulam theorem, The Bisection theorem, Deformation Retracts and Homotopy type, Homotopy invariance.	**CLO1** **CLO2** **CLO3** **CLO6**
III **15 Hours**	Direct sums of Abelian Groups, Free products of groups, Uniqueness of free products, Least normal subgroups, Free groups, Generators and relations, The Seifert-Van Kampen theorem, The Fundamental group of a wedge of circles.	**CLO4** **CLO6**
IV **15 Hours**	Classification of covering spaces: Equivalence of covering spaces, The general lifting lemma, The universal covering space, Covering transformation, Existence of covering spaces.	**CLO5** **CLO6**

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. M. A. Armstrong, *Basic Topology,* UTM, Springer, 2000.
2. S. Deo, *Algebraic Topology: A Primer (Texts and Readings in Mathematics),* Hindustan Book Agency, 2003.
3. M. J. Greenberg and J. R. Harper, *Algebraic Topology:* A First Course, CRC Press; 1st edition, 2018.
4. A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2009.
5. W. S. Massey, *A Basic Course in Algebraic Topology,* SPRINGER (SIE), 2007.
6. J. R. Munkres, *Elements of Algebraic Topology*, Perseus Books, 1995.
7. J. J. Rotman, *An Introduction to Algebraic Topology*, *Text in Mathematics, No. 119*, Springer, New York, 2004.
8. E. H. Spanier, *Algebraic Topology (2nd edition*), Springer-Verlag, New York, 2000.

Course Title: Riemannian Geometry

L	T	P	Credits
4	0	0	4

Course Code: MAT.710
Total Hours: 60

Learning outcomes:
The students will be able to

CLO1: learn basic concepts of Riemannian metric and Riemannian connection.
CLO2: Compute gradient, divergence, covariant derivatives, and analyze curves using geodesics, parallel transport, and the exponential map.
CLO3: Evaluate Lie derivatives of scalars, vectors, tensors, and connections, and apply commutation formulas to geometric problems.
CLO4: analyze curvature tensors and apply variational formulas and Jacobi fields to evaluate the global geometric structure of manifolds.
CLO5: utilize the Second Fundamental Form and the Gauss-Codazzi-Ricci equations to characterize the geometry of Riemannian submanifolds.

Units/Hours	**Content**	**Mapping with CLO**
I 16 Hours	Riemannian Metrics and Riemannian manifolds, Musical Isomorphisms, Affine connections, Covariant derivative and parallel transport, Covariant derivative of tensor fields, Levi-Civita connection and Pullback connections, Fundamental Theorem of Riemannian Geometry.	CLO1 CLO2
II 16 Hours	Geodesics, Geodesic flow, Exponential Map, Normal Coordinates, Gauss Lemma, Minimizing properties of Geodesics, Convex Neighborhoods. Tensors on Riemannian Manifolds, Curvature tensor, Sectional Curvature, Ricci and Scalar Curvatures, Weyl Tensor, Curvature of conformally related metrics.	CLO2 CLO4
III 14 Hours	Differential operators on Riemannian manifolds: Gradient vector fields, Divergence of a vector field, Laplacian operator, Lie derivative of a tensor field with respect to a vector field. Conjugate points, Cut locus, Introduction to Comparison Theory. The Jacobi Equation, Jacobi fields. First and Second variation formulas for length and energy.	CLO2 CLO3 CLO4
IV 14 Hours	Isotropic and Isometric immersions, Riemannian submanifolds, The Second Fundamental Form, Gauss formula, Weingarten	CLO5

	equation. Fundamental Equations: Gauss, Codazzi, and Ricci equations.	

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. J. M. Lee, Introduction to Riemannian Manifolds-176, GTM, Springer, 2021.
2. Manfredo P. Do Carmo and Francis Flaherty, Riemannian Geometry: Theory and Applications, Birkhausker Boston, 2011.
3. Kobayashi & Nomizu, Foundations of Differential Geometry, Vol.I, Wiley Edition 1996
4. John M. Lee, Riemannian manifolds: an introduction to curvature. Springer Science & Business Media, 2006.
5. B. O' Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, New York, 1983.

Course Title: Riemann-Finsler Geometry

L	T	P	Credits
4	0	0	4

Course Code: MAT.711

Total Hours: 60

Learning Outcomes: The objective of this course is to enable the students, concepts of Riemann-Finsler geometry so that they can pursue research in this area.

Learning outcomes:

The students will be able to

CLO1: learn basic concepts of Riemann-Finsler geometry.

CLO2: appreciate the topics covered in allied courses like Riemannian geometry, Mathematical Physics and their applications in allied areas.

CLO3: adequately prepared for pursuing research in Riemann-Finsler geometry.

CLO4: pursue further studies on Homogeneous Finsler Spaces.

Units/Hours	Content	Mapping with CLO
I 14 Hours	Minkowski norms, Euler's theorem, Fundamental inequality and its interpretation, Finsler structures: definitions and conventions, Examples: Minkowski and locally Minkowski spaces, Riemannian manifolds, Randers spaces, Berwald spaces, Finsler spaces of constant flag curvature. Fundamental metric tensor and Cartan tensor.	CLO1 CLO2 CLO3 CLO4
II 16 Hours	Vector bundle, Nonlinear connection on slit tangent bundle, Chern connection, Structure equations, Horizontal and vertical covariant derivatives, hh-, hv-, vv-curvatures, Bianchi identities and their consequences, Ricci identities, Geodesic spray coefficients, Flag curvature: Definition, example and its predecessor, Schur's lemma.	CLO1 CLO2 CLO3 CLO4
I 16 Hours	Rund's differential equation and its consequence, Criterion for strong convexity, Berwald frame, Moore frame, Geometrical setup on sphere bundle, Cartan scalar(I), Landsberg scalar(J) and Gaussian curvature(K), Riemannian arc length of indicatrix, Gauss Bonnet theorem for Landsberg surfaces.	CLO1 CLO2 CLO3 CLO4
I 14 Hours	Geodesics: Sprays, shortest paths. Projectively equivalent Finsler metrics, Projectively flat metrics, Parallel vector fields, Parallel translations, Berwald metrics, Landsberg metrics, Distortion and S-curvature, Randers metric of isotropic S-curvature.	CLO1 CLO2 CLO3 CLO4

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. P. L. Antonelli (ed.), *Handbook of Finsler Geometry*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
2. D. Bao, S. S. Chern, Z. Shen, *An Introduction to Riemann Finsler Geometry, Graduate texts in Mathematics 200*, Springer-Verlag, New York, 2012.
3. Xinyue Cheng and Zhongmin Shen, *Finsler geometry-An Approach via Randers spaces*, First Edition, Springer Berlin Heidelberg, 2013.
4. S. S. Chern and Z. Shen, *Riemann-Finsler Geometry,* Nankai Tracts in Mathematics, Vol. 6. World Scientific Publishing Co. Pvt. Ltd., 2005.
5. M. Matsumoto, *Foundations of Finsler Geometry and Special Finsler Spaces*, Kaisheisha press, Saikawa, Otsu, 520, Japan, (1986).
6. Z. Shen, *Lectures on Finsler geometry*, World Scientific Press, 2001.
7. Y. B. Shen and Z. Shen, *Introduction to Modern Finsler Geometry*, World Scientific Publishing Co. Pvt. Ltd., 2016.
8. H. Rund, The Differential Geometry of Finsler Spaces:101, Springer-Verlag, 2012.

Course Title: General Relativity

Course Code: MAT.712

Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning Outcomes:

The students will be able to

CLO 1:. learn basic concepts of Tensor Algebra and Riemannian metric.

CLO 2:. understand the concept of Special theory of relativity.

CLO 3:. obtain the spherically symmetric solutions of Einstein field equations

CLO 4:. learn the large scale structure of the universe by studying cosmology.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Transformation of coordinates, Tensor Algebra, Smooth manifolds: Definition and examples, vector fields, Lie brackets and Lie derivatives. Riemannian metric, parallel transport, covariant derivative, affine connection, Riemannian connection, Geodesics, Riemann curvature tensor and its symmetric properties, Ricci tensor, Bianchi identities, Einstein tensor.	CLO1
II 15 Hours	Postulates of Special Theory of Relativity, Lorentz Transformation and consequences, Minkowski Diagram, Four dimensional spacetime continuum, Four vector formulation. Transition of Special to General Theory of Relativity, Principle of covariance, Equivalence Principle and consequences, Stress Energy Tensor, Einstein Field Equations and Newtonian limit.	CLO2
III 15 Hours	Spherically symmetric solution to Einstein Field Equation in free space and in matter, Schwarzshild line element. Schwarzschild Black Holes. Equation of Planetary Orbits, Crucial tests of General Theory of Relativity, Advance of Perihelion, Gravitational bending of light and Gravitational Redshift.	CLO3
IV 15 Hours	Cosmology: Large scale structure of Universe, Galactic Densities and the darkness of the	CLO4

	Night Sky, Galactic Number Counts, Olber's paradox, Cosmological principles, Relativistic Universe and models. Einstein and de-Sitter models of static universe, Dynamical Universe, Comoving time, Red Shifts and Horizons, Friedmann-Robertson-Walker line element, Open and Closed Universe, Hubbles law, Early Universe.	

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings

1. R. Adler, M. Bazin and M. Schiffer, Introduction to General Relativity, McGraw Hill, 1965.
2. S. Caroll, Spacetime and geometry: an introduction to general relativity, Addison Wesley, 2004.
3. J. B. Hartle, Gravity: an introduction to Einstein's general relativity, Pearson education, 2003.
4. R. D. Inverno, Introducing Einstein's relativity, Oxford university press, 2005.
5. C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation, W. H. Freeman and Co. 1973.
6. J. V. Narlikar, General Relativity and Cosmology, Macmillan, 1978.
7. A. S. Ramsey, Newtonian Attraction, Cambridge University Press, 1964.
8. B. F. Schutz, A First Course in General Relativity, Cambridge University Press, 2012.
9. S. Weinberg, Gravitation and cosmology: principles and applications of the general theory of relativity, John wiley and Sons, 2004.

Course Title: Mathematical Modelling in Biological Systems

Course Code: MAT.713
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

The students will be able to

CLO1. Understand the basic concepts, mathematical modeling in biological systems.
CLO2. Create models for real life problems.
CLO3. Learn the technique of solving the difference equations.
CLO4. learn the application of various epidemic models.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Introduction to mathematical models. Stability of linear and nonlinear systems. Phase Plane method and qualitative solutions. Continuous Models: growth models, harvesting model, delay models, age distribution models, interaction populations models	CLO1
II 15 Hours	Models for molecular events. Limit cycles, oscillations and excitable systems. May's Model, ratio dependent model of two interacting species, two prey-one predator system with ratio-dependent.	CLO2
III 15 Hours	Discrete dynamics: linear models, graphical solutions of difference equations, equilibrium analysis, periodic-doubling and chaotic behavior, system of two difference equations. Food chain and Food web models. Stage-structured models. Introduction to Modelling Epidemics: Plague, Measles, T.V., Cancer, Malaria	CLO3
IV 15 Hours	Simple Epidemics Models: SI, SIS, SIR,SIRS, SEIS, SEIR. Stability of simple epidemics models. Models for disease with no immunity. Compartment Models in Epidemiology. Models for demographic effect, Disease as population control, Infective period of fixed length. Impulsive control in continuous and Discrete systems. Pulse vaccination epidemic models.	CLO4

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. S.A. Levin, Frontiers in Mathematical Biology, Springer-Verlag, 1994.
2. Nicholas F. Britton, Essential Mathematical Biology, Springer, 2002.
3. Leach Edelstein-Keshet, Mathematical Models in Biology, SIAM's Classics in Applied Mathematics, 1988.
4. J.D. Murray, Mathematical Biology, Springer (3^{rd} Ed.), 2002.
5. Fred Brauer and Carlos Castillo-Chavez, Mathematical Models in Population Biology and Epidemiology.
6. Mark Kot, Elements of Mathematical Ecology, Cambridge University Press, 2001.
7. N.T.J. Bailey, The mathematical theory of infectious diseases and its applications. 2nd edition, 1975.

Course Title: Qualitative Theory of Dynamical Systems

Course Code: MAT.714

Total Hours: 60

L	T	P	Credits
4	0	0	4

The students will be able to

CLO1. know about dynamical Systems from an applied and practical point of view.
CLO2. how to compute the behavior of differential equations as parameters vary.
CLO3. know about the techniques that include bifurcation analysis and computation of normal forms, geometric methods, and the method of averaging.
CLO4. To understand the concept of Local and Global bifurcations.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	**Basic Concepts**: Discrete and continuous dynamical systems. Linear and nonlinear systems and principle of superposition. Linear and nonlinear forces. Concepts of evolution, iterations, orbits, fixed points, periodic and aperiodic (chaotic) orbits. Basics of Linear Algebra: Symmetric & Skew-symmetric matrices, matrix norm and singular value decomposition. Eigenvalues, eigenvectors, and dynamical interpretation. **Activity:** Students will apply some basics of linear algebra in study of dynamical systems.	CLO1
II 15 Hours	Canonical forms; simple and non-simple canonical systems. System of Equations. **Stability Analysis**: Stability of a fixed point and classification equilibrium states (for both discrete and continuous systems). Concept of bifurcation and classification of bifurcations. Concepts of Lyapunov stability & Asymptotic stability of orbits.	CLO2

	Activity: Students will check the stability of some dynamical systems by using the concept of bifurcation.	
III 15 Hours	Phase Portraits of various Linear and Nonlinear systems. Hopf bifurcation. Concept of attractors and repellers, limit cycles and torus. **Phenomena of Bifurcation**: Definition of bifurcation. Bifurcations in one, two and higher dimensional systems. **Activity:** Students will draw the phase portraits of some dynamical systems using some software.	CLO3
IV 15 Hours	Hopf, Period doubling, Saddle node, Transcritical bifurcations. Feigenbaum's number. Local and Global bifurcations. Homoclinic & Hetero-clinic points and orbits. Poincaré-Bendixson Theorem. Conservative and Dissipative Systems. **Activity:** Students will draw the phase portraits of some dynamical systems using some software and will study the orbits.	CLO4

Suggested Readings:

1. Differential Equations and Dynamical Systems by Lawrence Perko, Springer-Verlag, 2006.
2. Differential Equations, Dynamical Systems and an Introduction to Chaos by Morris W. Hirsch, Stephen Smale and Robert L. Devaney, Academic Press, 2013
3. Dynamical Systems and Numerical Analysis by A.M. Stuart and A.R. Humphries, Cambridge University Press, 1998.
4. Fractals Everywhere by M. F. Barnsley, 2nd edition, Academic Press, 1995
5. An introduction to chaotic dynamical system by Robert L. Devaney, Addison Wesley publishing house Co inc. 1989.
6. Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering, by Steven H. Strogatz, Westview Press.
7. A. F. Beardon, Iteration of rational functions, Springer Verlag , New York, 1991.
8. S. Morosawa, Y. Nishimura, M. Taniguchi, T. Ueda, Holomorphic dynamics, Cambridge University Press, 2000.

Course Title: Advance Linear Algebra

L	T	P	Credits
4	0	0	4

Course Code: MAT.715

Total Hours: 60

Learning outcomes:

The students will be able to

CLO1: Review the basic notions in linear algebra that are often used in mathematics and other sciences

CLO2: Define Vector spaces, Subspaces and related results.

CLO3: Define Linear transformations and characteristic polynomials with examples.

CLO4: Illustrate various properties of canonical forms.

CLO5: Study of inner product spaces.

CLO6: Explain concepts of the Gram-Schmidt orthogonalization process and its application in construction of orthonormal basis.

Unit/ Hours	Content	Mapping with CLO
I 14 Hours	Vector spaces, Subspaces: Definition and Examples, Linear dependence and independence, Basis and dimensions, Coordinates, Linear transformations, Algebra of linear transformations, Isomorphism, Matrix representation of a linear transformation.	**CLO1 CLO2**
II 16 Hours	Change of basis, Rank and nullity of a linear transformation. Linear functionals, Dual spaces, Transpose of a linear transformation. Annihilating Polynomials: Characteristic polynomial and minimal polynomial of a linear transformation, Characteristic values and Characteristic vectors of a linear transformation, Cayley Hamilton theorem.	**CLO3**
III 16 Hours	Diagonalizing matrices, Diagonalizing real symmetric matrices, Characteristic polynomials and minimal polynomials of block matrices, Canonical forms: Jordan canonical forms, rational canonical forms. Quotient spaces, Bilinear forms, Symmetric and skew-Symmetric bilinear forms, Sylvester's theorem, quadratic forms, Hermitian forms.	**CLO4**
IV 14 Hours	Inner product spaces. Norms and distances, Orthonormal basis, Orthogonality, Schwarz inequality, The Gram-Schmidt orthogonalization process.Orthogonal and positive definite matrices.	**CLO5 CLO6**

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. J. Gilbert and L. Gilbert, *Linear Algebra and Matrix Theory,* Cengage Learning, 2004.
2. K. Hoffman and R. Kunze: *Linear Algebra,* 2nd Edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India, 2004.
3. V. Bist and V. Sahai, *Linear Algebra,* Narosa, Delhi, 2002.
4. S Lang, Linear Algebra, Undergraduate texts in mathematics, Springer, 1989.

Course Name: Stochastic Differential Equations
Course Code: MAT.718
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

The students will be able to

CLO1: Explain Stochastic Processes.
CL02: learn and solve the Stochastic differential equations(SDE).
CL03: find the connections between connection between SDE and partial differential equations (PDE)
CL04: formulate the SDE models.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	A review of the necessary prerequisites in probability theory, including an introduction to stochastic processes in discrete and continuous time. Brownian motion (the Wiener process), the Ito integral and the Ito calculus,	CLO1
II 15 Hours	Introduction to stochastic differential equations (SDE) and solving certain types of SDE analytically with Ito calculus. Furthermore the general existence and uniqueness theory for SDE, numerical methods for simulating solutions to SDEs.	CLO2
III 15 Hours	The connection between SDE and partial differential equations (PDE) :(e.g. Fokker-Plancks's equation) solutions of PDEs in separate points by using simulations of SDEs.	CLO3
IV 15 Hours	SDE models are formulated and fitted to given data, and are studied in some examples of applications.	CLO4

Recommended Books

1. I. Karatzas and S. E. Shreve, Brownian motion and stochastic calculus, 2nd ed., Graduate Texts in Mathematics, vol. 113, Springer-Verlag, New York, 1991
2. B. Øksendal, Stochastic differential equations, 6th ed., University text, Springer-Verlag, Berlin, 2003.
3. A. Friedman, Stochastic differential equations and applications, Dover Publications Inc., Mineola, NY, 2006.
4. Allen E., Modeling with Itô stochastic differential equations Springer, 2007

Course Title: Fluid Dynamics
Course Code: MAT.719
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

The students will be able to

CLO1: Review of Basic Concepts of Fluid Dynamics.
CL02: learn equations of motion of fluid.
CL03: Will be able to solve many problems related to fluid dynamics.
CL04: learn basics of Navier-Stokes equations.

Students can model basic and some advanced fluid flow problems and solve them.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Review of gradient, divergence and curl. Elementary idea of tensors. Velocity of fluid, Streamlines and path lines, Steady and unsteady flows, Velocity potential, Vorticity vector, Conservation of mass, Equation of continuity.	CLO1
II 15 Hours	Equations of motion of a fluid, Pressure at a point in fluid at rest, Pressure at a point in a moving fluid, Euler's equation of motion, Bernoulli's equation. Singularities of flow, Source, Sink, Doublets, Rectilinear vortices.	CLO2
III 15 Hours	Complex variable method for two-dimensional problems, Complex potentials for various singularities, Circle theorem, Blasius theorem, Theory of images and its applications to various singularities. Three dimensional flow, Irrotational motion, Weiss's theorem and its applications.	CLO3
IV 15 Hours	Viscous flow, Vorticity dynamics, Vorticity equation, Reynolds number, Stress and strain analysis, Navier-Stokes equation, Boundary layer Equations.	CLO4

Suggested Readings:

1. G K Batchelor, *An Introduction to fluid dynamics*, Cambridge University Press , A seminal book on fluid dynamics
2. F M White, *Fluid Mechanics*, Tata McGraw Hill , One will find many solved and unsolved problems.

3. F. Chorlton, *A text book of fluid dynamics*, Von Nostrand Reinhold/CBS , Very good book for beginners.
4. L. M. Milne Thomson, *Theoretical Hydrodynamics*, Macmillan and Co , A good book on mathematical fluid dynamics

Course Title: Advanced Topics in Topology and its Applications
Course Code: MAT. 720
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

The students will be able to

CLO1: Understand basic structures and properties of uniform spaces.
CL02: Explore topologies on function spaces and their key properties.
CL03: Learn applications of topology in science and technology.
CL04: Explore topological aspects of graphs, including planarity, embeddings, and applications to circuit design.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	**Uniform Spaces:** Diagonal Uniformities, surroundings, examples; usual, metric, discrete and trivial uniformities, uniform topology, uniform isomorphism, uniform covers, base and sub-base for a covering uniformities, metrizability of a uniformity.	CLO1
II 15 Hours	**Function Spaces:** The compact-open topology, on Y^X, Hausdorffness and regularity of Y^X, Continuity of composition; the Evaluation Map. Cartesian Products, Application to Identification Topologies; Theorem of Whitehead, Basis of Z^Y and Compact Subsets of Z^Y. Equicontinuity and Arzela - Ascoli Theorem. Comparison of topologies on Z^Y.	CLO2
III 15 Hours	**Applications of the basic Concepts**: Intersection Value, Regularly closed sets and application to Geographic Information Systems (GIS). Quotient Spaces, Examples of quotient spaces with application to Configuration and Phase Spaces. Application of Metrics to Comparison of DNA sequences; the Levenshtein distance. Application of Connectedness to Automated Guided Vehicles.	CLO3
IV 15 Hours	**Graphs and Topology**: Introduction, Complete Bipartite graph, Graph Isomorphism. Graph Embeddings: Planar Graphs, Kuratowski's theorem, Euler's formula for Planar Graphs. Crossing Number and thickness of a graph, An Application to Electronic Circuit Design.	CLO4

Suggested Readings:

1. Colin Adams, Robert Franzosa: Introduction to Topology Pure and Applied. Pearson Education Inc; 2008.
2. Stephen Willard, General Topology, New York: Dover Publications, Inc., 2004.
3. James Dugundji, Topology, Universal Book Stall, New Delhi, 1990.
4. R. Munkres, Topology, Second Edition, Pearson India Education services Pvt. Ltd., 2015.

Course Title: Stochastic Processes and its Applications
Course Code: MAT.721
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:
The students will be able to
CLO1: Understand the theoretical foundations of stochastic processes and their classifications.

CLO2: Analyze discrete-time and continuous-time Markov processes and their long-term behavior.

CLO3: Apply Poisson and birth–death processes in modeling real-world systems.

CLO4: Develop and analyze queueing and renewal models.

CLO5: Evaluate advanced stochastic models and their applications in engineering, communication systems, and finance.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Introduction to stochastic processes: definition, classification, and state space; discrete-time stochastic processes; Markov chains; transition probability matrices; Chapman–Kolmogorov equations; classification of states; steady-state and limiting behavior.	CLO1
II 15 Hours	Continuous-time Markov processes: Poisson process and its properties; birth–death processes; infinitesimal generator matrix; Kolmogorov forward and backward equations; transient and steady-state analysis.	CLO2
III 15 Hours	Queueing theory: structure of queueing systems; Kendall's notation; M/M/1, M/M/1/N models; Little's Law; waiting time and response time distributions; renewal processes and renewal theory; applications in service and communication systems.	CLO3
IV 15 Hours	Advanced stochastic models: Markov renewal processes; semi-Markov processes; Markov regenerative processes;	CLO4 & CLO5

	introduction to martingales; applications in reliability engineering, finance, and stochastic modeling.	

Suggested Readings:

1. L. B Castaneda, V. Arunachalam, S. Dharmaraja, *Introduction to Probability and Stochastic Processes with Applications*, Wiley, Hoboken, NJ, USA, 2012.
2. J. Medhi J., *Stochastic Processes* (3rd Ed.), New Age International Publishers, 2009.
3. A K Basu, *Introduction to Stochastic Proces*s, Narosa Publishing House, 2003.
4. S Dharmaraja, *Introduction to Stochastic Processes: Queues, Finance, and Credit Risk*, Springer, 2025.
5. S. M. Ross, *Stochastic Processes*, 2nd edition, John Wiley, 1995.

Course Title: Mathematical Control Theory
Course Code: MAT. 722
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

The students will be able to

CLO1: Apply the Kalman decomposition and rank conditions to determine the controllability and observability of linear differential systems in state-space representation.

CLO2: Assess the stability of linear systems and establish the mathematical relationship between stabilizability and controllability for both linear and nonlinear differential equations.

CLO3: Employ the theory of semigroups of operators and the Hille–Yosida theorem to provide integral representations of linear systems and analyze the dynamics of delay systems.

CLO4: Construct a rigorous mathematical framework for infinite-dimensional systems using the properties of images and kernels of linear operators.

CLO5: Critically differentiate between various levels of controllability, including exact, approximate, and null controllability, within infinite-dimensional spaces.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Linear differential equations, Controllability matrix, Rank condition, Classification of control systems, Kalman decomposition, Observability of linear systems	CLO1
II 15 Hours	Stable linear systems, Stabilizability and its relation with controllability, Nonlinear differential equations, Controllability of nonlinear systems	CLO2
III 15 Hours	Semigroups of operators, Hille–Yosida theorem, Integral representation of linear systems, Delay systems	CLO3
IV 15 Hours	Infinite-dimensional systems: basic framework, Images and kernels of linear operators, Controllability operator in infinite-dimensional spaces, Concepts of controllability (exact, approximate, null controllability), Duality between controllability and observability	CLO4, CLO5

Suggested Readings:

1. Zabczyk, Jerzy. *Mathematical control theory*. Springer International Publishing, 2020.
2. Curtain, Ruth F., and Hans Zwart. *An introduction to infinite-dimensional linear systems theory*. Vol. 21. Springer Science & Business Media, 2012.

Course Name: Lie Algebras
Course Code: MAT.723
Course Type:
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

CLO1: Identify and describe the basic structure, subalgebras, ideals, homomorphisms, and automorphisms of Lie algebras.
CLO2: Apply criteria and theorems to determine solvability, nilpotency, and semi-simplicity of Lie algebras.
CLO3: Demonstrate understanding of Lie algebra modules, inner derivations, Jordan-Chevalley decomposition, and preservation under representations.
CLO4: Analyze representations of sl(2, C), compute weights and highest weight vectors, and classify irreducible modules.
CLO5: Examine abstract root systems, determine Weyl groups, root strings, and classify rank 2 root systems through structural properties.

Units/Hours	Contents	Mapping with CLO
I 14 Hours	Definition and examples of Lie algebras, classical Lie algebras, derivations of a Lie algebra, abelian Lie algebra, Lie subalgebras, ideals and homomorphisms, normalizers and centralizers of a Lie subalgebra.	CLO1
II 16 Hours	Representations of lie algebras (definition and some examples), automorphisms of a Lie algebra. Solvable algebra, Solvable radical, nilpotent algebra, Engel's Theorem, semi-simple Lie algebra, Lie's Theorem, Jordan-Chevalley decomposition (existence and uniqueness), Cartan trace criterion for solvability.	CLO2
III 16 Hours	Killing form and criterion for semisimplicitySimple ideals, inner derivations, abstract Jordan-Chevalley decomposition, Lie algebra modules, Schur's Lemma, Casimir elements of a representation. Weyl's Theorem for preservation of Jordan-decomposition,	CLO3 CLO4
IV 14 Hours	Representation of sl(2, C), weights, highest weight, maximal vectors. Classification of irreducible modules, toral and maximal toral-subalgebra, root space decomposition, orthogonality properties and integrality	CLO4 CLO5

	properties of roots.	

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning**.**

Suggested Readings:

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Text in Mathematics, 9, Springer-Verlag, 1980.
2. N. Jacobson, *Lie Algebras,* Wiley-Interscience, New York, 1962.
3. J. P. Serre, *Lie Algebras and Lie Groups,* Benjamin, New York, 1965.
4. N. Bourbaki, *Lie Groups and Lie Algebras*, Springer Science & Business Media, 1998.
5. K. J. Alexander, *An Introduction to Lie Groups and Lie Algebras*, Cambridge University Press.
6. S. Kumaresan, *Differential Geometry and Lie Groups*, Hindustan Book Agency.
7. B. Hall, *Lie Groups, Lie Algebras and Representations: An Elementary Introduction*, Second Edition, Springer.
8. P. J. Olver, *Application of Lie Groups to Differential Equations*, Second Edition, Springer.

Course Title: Game Theory and its Applications
Course Code: MAT.724
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

The students will be able to

CLO1: Understand the foundational concepts of strategic games and equilibrium analysis.

CLO2: Analyze static and dynamic games using Nash and subgame perfect equilibria.

CLO3: Evaluate games with incomplete information using Bayesian frameworks.

CLO4: Apply cooperative and mechanism design principles in economic and engineering systems.

CLO5: Investigate advanced topics including repeated, stochastic, and algorithmic games.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Introduction to strategic games: definition, players, strategies, payoffs; normal form games; dominance; best response; Nash equilibrium; mixed strategies; existence of equilibrium.	CLO1
II 15 Hours	Dynamic and extensive form games: game trees; subgame perfect equilibrium; backward induction; repeated games; Folk theorem.	CLO2
III 15 Hours	Games with incomplete information: Bayesian games; types and beliefs; Bayesian Nash equilibrium; signaling games; Perfect Bayesian equilibrium.	CLO3
IV 15 Hours	Cooperative games and mechanism design: core, Shapley value; bargaining; auctions; incentive compatibility; algorithmic and stochastic games; applications.	CLO4 & CLO5

Suggested Readings:

1. Martin J. Osborne, and Ariel Rubinstein, *A course in game theory*. MIT press, 1994.
2. D. Fudenberg, and J. Tirole, *Game theory*. MIT press, 1991.
3. N. Nisan, T. Roughgarden, E. Tardos, and V.V. Vazirani, *Algorithmic game theory*. Cambridge university press, 2007
4. S. Tadelis, *Game theory: an introduction*. Princeton university press, 2013.

Course Name: Complex Manifolds
Course Code: MAT.725
Total Hours: 60

L	T	P	Credits
4	0	0	4

Learning outcomes:

After successful completion of this course, students will be able to:
CLO1: Explain complexification of real vector spaces, identify complex structures, and provide examples of complex manifolds.
CLO2: Describe tangent and cotangent spaces, define complex structures on these spaces, and work with vectors, one-forms, and tensor fields on complex manifolds.
CLO3: Analyze almost complex structures, compute the Nijenhuis tensor, and determine integrability conditions for complex structures.
CLO4: Apply Hermitian and Kaehlerian structures to vector spaces and manifolds, compute curvature tensors, and evaluate holomorphic sectional curvature.
CLO5: Understand nearly Kaehler and para Kaehler manifolds, describe projective correspondences, examine conformal flatness, and verify related curvature identities.

Units/Hours	Contents	Mapping with CLO
I 15 Hours	Complexification of a real vector space, complex structure, relation between complexification and complex structure, conjugate complex structure, complexification of the dual space, expressions in terms of bases, orientations, complex structures, necessary conditions for a complex structure to exist, examples of complex manifolds.	CLO1
II 15 Hours	The tangent space and the cotangent space, complexified tangent space, complex structure on the tangent space, complex structure on the cotangent space, relation between the canonical complex structure and the manifold complex structure, vectors and tensors, real tensors, vectors and one-forms of type (1,0) and type (0,1), complex tensors and complex manifolds, tensor fields.	CLO2
III 15 Hours	Almost complex structure, conditions for existence of an almost complex structure, almost complex structure on a complex manifold, the Nijenhuis tensor, vanishing of the Nijenhuis tensor as necessary and sufficient condition for integrability. Hermitian structures on vector spaces, Hermitian manifolds, curvature tensor on a Hermitian manifold, holomorphic sectional curvature.	CLO3 CLO4
IV 15 Hours	Kaehlerian manifolds, curvature on Kaehlerian manifolds, complex space forms. Nearly Kaehler and para Kaehler Manifolds, projective correspondence between two nearly Kaehler manifolds, conformal flatness of a para Kaehler manifold, curvature identities.	CLO5

Suggested Readings:

1. S.S. Chern, W. H. Chen and K. S. Lam, Lectures on Differential Geometry, World Scientific, 2000.
2. E.J. Flaherty, Hermitian and Kaehlerian Geometry in Relativity, LNP 46, Springer, 1976.
3. Kobayashi and Nomizu, Foundations of Differential geometry, Vol-II, Interscience Publishers, 1963.
4. J. M. Lee, Introduction to Complex Manifolds, GTM 244, AMS, 2024.
5. K. Yano & M. Kon, Structures on Manifolds, World Scientific, 1984.

Course Name: Contact Manifolds
Course Code: MAT.726
Total Hours: 60

L	T	P	Credits
4	0	0	4

Course learning outcomes:

After successful completion of this course, students will be able to:

CLO1: Define contact manifolds, almost contact manifolds, and related tensor fields, and analyze curvature properties of contact metric manifolds.

CLO2: Characterize K-contact manifolds, compute their sectional curvature, and understand locally symmetric K-contact manifolds.

CLO3: Describe Sasakian manifolds, compute φ-sectional curvature, and analyze Sasakian space forms and cosymplectic manifolds.

CLO4: Examine cosymplectic, nearly cosymplectic, and trans-Sasakian manifolds, including curvature properties and 3-dimensional examples.

CLO5: Understand para-contact manifolds, determine integrability of almost para-contact structures, and study para-Sasakian and LP-Sasakian manifolds.

Units/ Hours	Contents	Mapping with CLO
I 15 Hours	Contact manifolds: Definition, and Examples, Almost contact manifolds, Killing vector field, The tensor field h, Curvature properties of contact metric manifolds	CLO1
II 15 Hours	K-contact Manifolds, Characterizations of K- contact manifolds, Curvature properties, Sectional curvature, Locally symmetric K- contact manifolds.	CLO2
III 15 Hours	Sasakian manifolds, Curvature properties, - sectional curvature of a Sasakian manifold, cosymplectic manifolds, Sasakian space forms, trans- Sasakian manifolds, 3- dimensional trans- Sasakian manifolds.	CLO3 CLO4
IV 15 Hours	Cosymplectic manifolds, Nearly cosymplectic manifolds, cosymplectic space forms, Para contact manifolds, Examples, Torsion tensor fields, Integrability of almost para contact structure, Para-Sasakian manifolds, LP- Sasakian manifolds.	CLO4 CLO5

Suggested Readings:

1. D. E. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Birkhauser, 2010.

2. K. Yano & M. Kon, Structures on Manifolds, World Scientific, 1984.